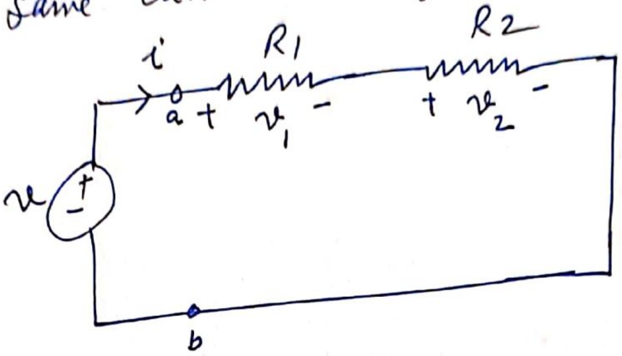
Voltage Divider Theorem :-

Consider a single loop circuit in which two resistors R_1 and R_2 are connected in series and a voltage v is applied across the series combination. Same current i flows through R_1 & R_2.



We have ohms law across R_1 & R_2 gives

$$v_1 = i R_1 \quad \text{and} \quad v_2 = i R_2$$

Now Apply KVL in loop;

$$+v - v_1 - v_2 = 0$$

or

$$v = v_1 + v_2$$
$$= i R_1 + i R_2$$
$$v = i(R_1 + R_2) \Rightarrow \boxed{v = i R_{eq}}$$

$$\therefore i_1 = \frac{v}{R_1 + R_2}$$

$$\therefore \boxed{v_1 = \frac{R_1}{R_1 + R_2} v} \quad \& \quad \boxed{v_2 = \frac{R_2}{R_1 + R_2} v}$$

Where $R_{eq} = R_1 + R_2$ = Equivalent series resistance.

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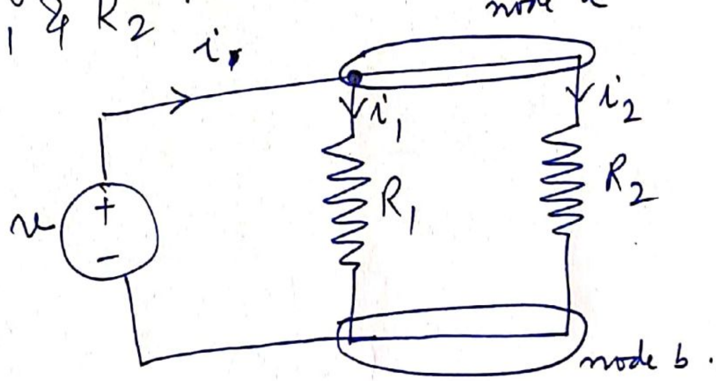
Thus the equivalent series resistance is simply the sum of the resistances connected in series.

Also the source voltage gets divided among the resistors in direct proportion to their resistances. the larger the resistance, the larger the voltage drop. This is called principle of voltage division or Voltage Divider Theorm.

Current Divider Theorm

Consider two resistances R_1 & R_2 connected in parallel and a voltage v applied across them. Being in parallel same voltage appears across R_1 & R_2.



$$v = i_1 R_1 = i_2 R_2$$

or $i_1 = \frac{v}{R_1}$; $i_2 = \frac{v}{R_2}$

Apply KCL at a; $i - i_1 - i_2 = 0$

$$i_1 = i_1 + i_2$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
$$= \frac{v}{R_{eq}}$$

where R_{eq} is the equivalent resistance of resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \; ; = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore \boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

The equivalent resistance of two parall resistances is equal to the product of their resistances divided by their sum.

now;

$$i_1' = \frac{v}{R_1} = \frac{i R_{eq}}{R_1} = \frac{\cancel{R_1} R_2}{(R_1 + R_2)\cancel{R_1}} \cdot i$$

$$\boxed{i_1' = \frac{R_2}{R_1 + R_2} i}$$

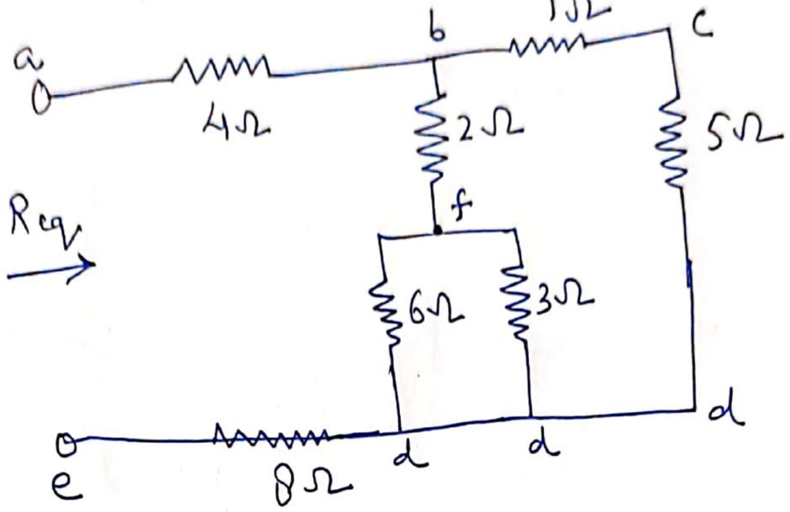
&

$$i_2' = \frac{v}{R_2} = \frac{i R_{eq}}{R_2} = \frac{R_1 \cancel{R_2}}{(R_1 + R_2)\cancel{R_2}} \times i'$$

$$\boxed{i_2' = \frac{R_1}{R_1 + R_2} i}$$

→ Current Division Rule

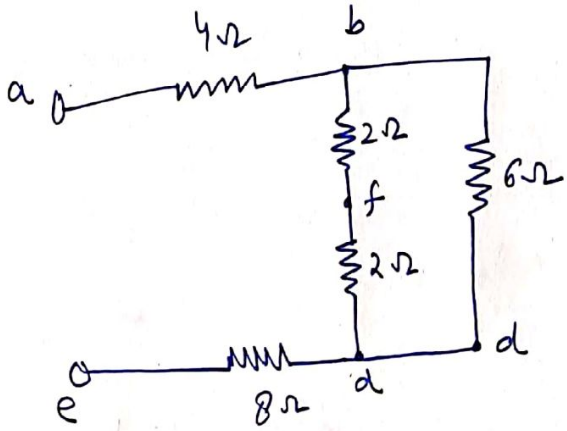
Example: Find Req for the circuit shown below;



6Ω & 3Ω in ||el $\rightarrow \frac{6\times3}{6+3} = \frac{18}{9} = 2\,\Omega$

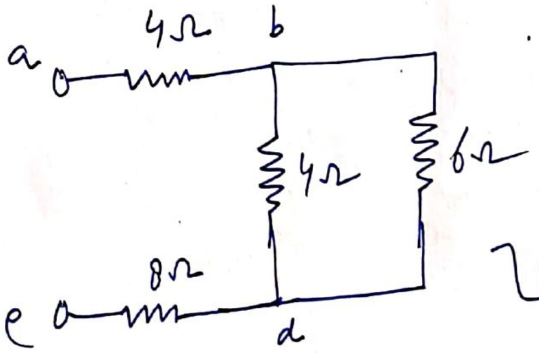
1Ω + 5Ω in series $\rightarrow$ $1+5 = 6\,\Omega$;

Draw eq. ckt $\rightarrow$



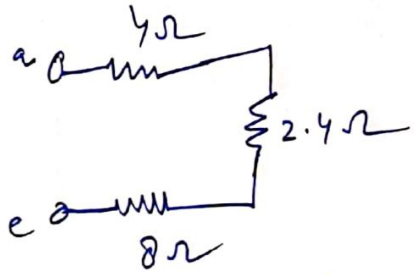
2Ω + 2Ω in series;

$=2+2 = 4\,\Omega$;



4Ω and 6Ω in parall

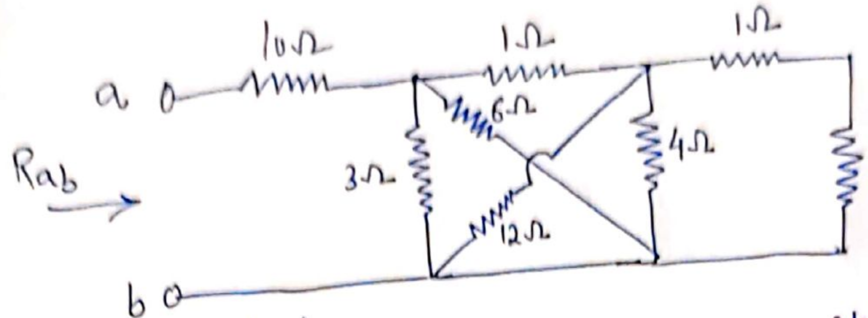
$\rightarrow \frac{6\times4}{6+4} = \frac{24}{10} = 2.4\,\Omega$.



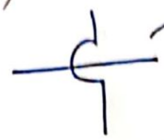
4Ω, 2.4Ω & 8Ω in series $\rightarrow$ $4+2.4+8 = 14.4\,\Omega$

$\boxed{R_{eq} = 14.4\,\Omega}$

Example 1:



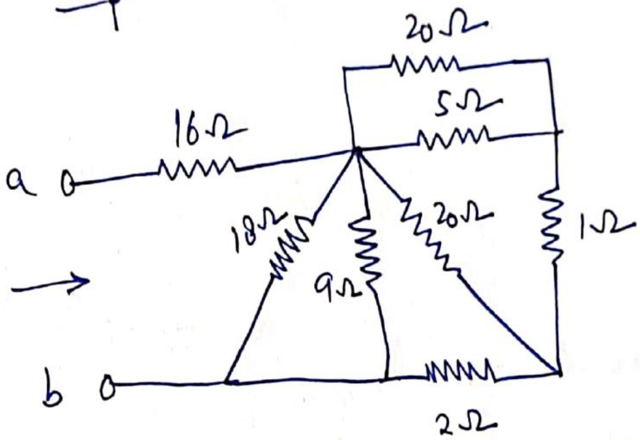
Find R_{ab} = ? 11.2Ω (Ans)

 → means no interconnection of two links.

 → means connection.

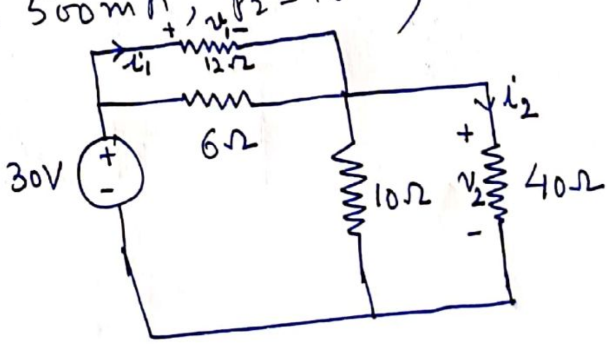
Example 2

Find R_{ab} = ? 19Ω (Ans) →



Example 3: Find v_1 and v_2 in the circuit shown below. Also calculate i_1 and i_2 and power dissipated in the 12 Ω and 40 Ω resistors.

(v_1 = 10V, v_2 = 20V, i_1 = 833.3mA, P_1 = 8.333W, i_2 = 500mA, P_2 = 10W)

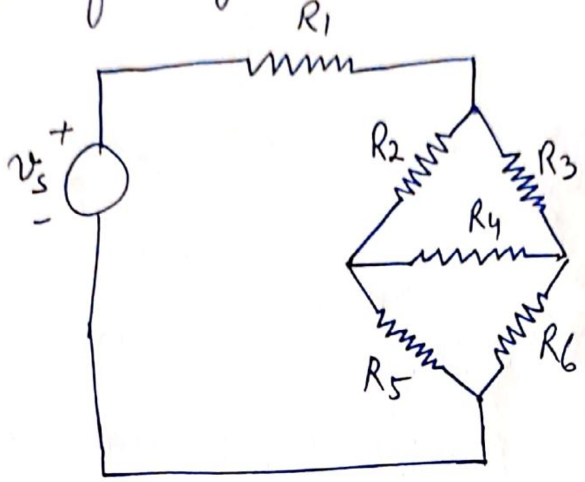


STAR- DELTA TRANSFORMATION

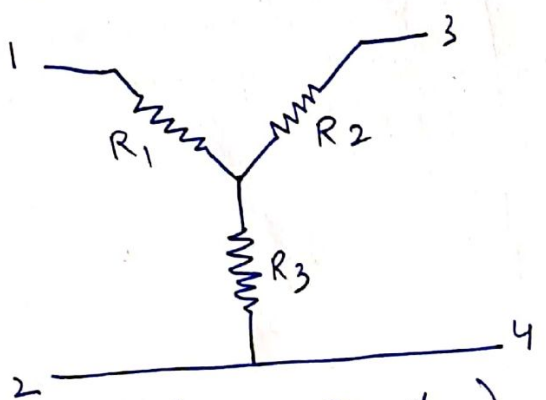
Need of Y-Δ Transformation

Situations often arise in circuit analysis when resistors are neither in Series nor in parallel. e.g.

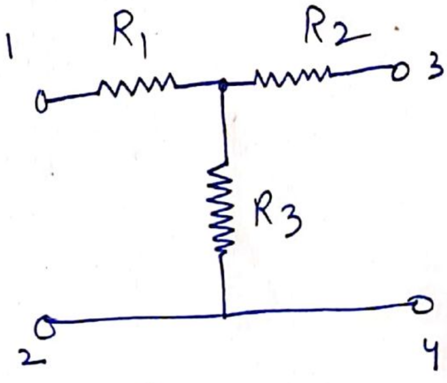
Consider the following circuit:



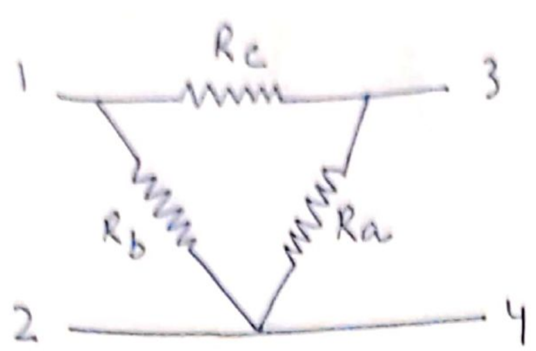
We can not find the equivalent resistance because R, R_2, R_3, R_4, R_5, R_6 are neither in Series nor parallel combination. Such type of circuits are simplified by use three terminal equivalent networks. These are WYE (Y) or T - network and delta (Δ) or PI(π) network.



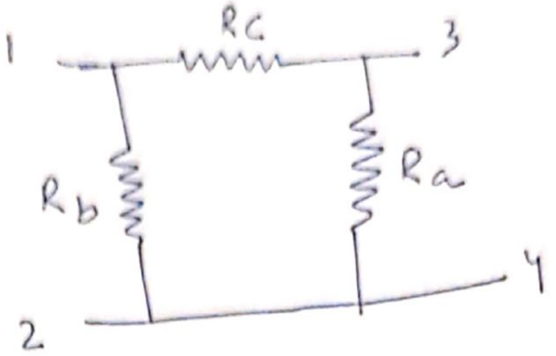
(Y N/W)



(T- N/W)

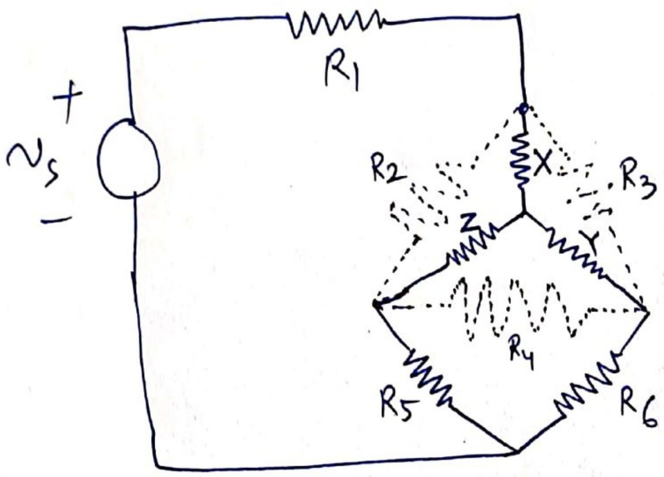


Δ Network.

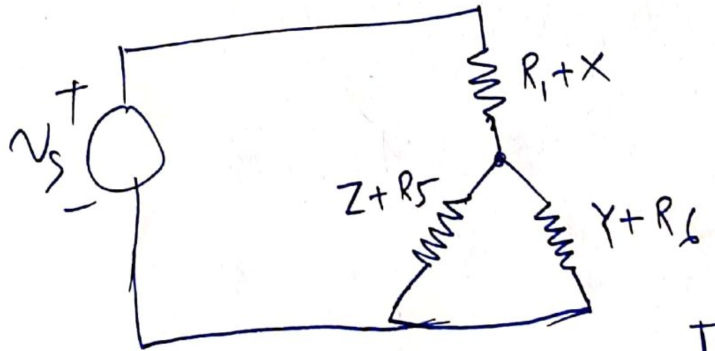


π - N/W.

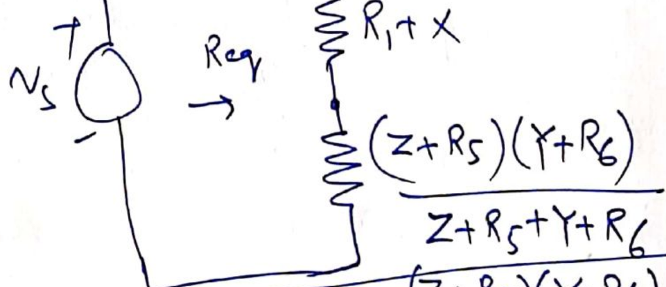
In the given circuit, if we convert Δ (upper Triangle into its equivalent Y connection, the circuit will become; (Dotted line & Resistor shows original circuit).



Now Z and R5 are in series, & Y and R6 are in series and R1 & X are in series.

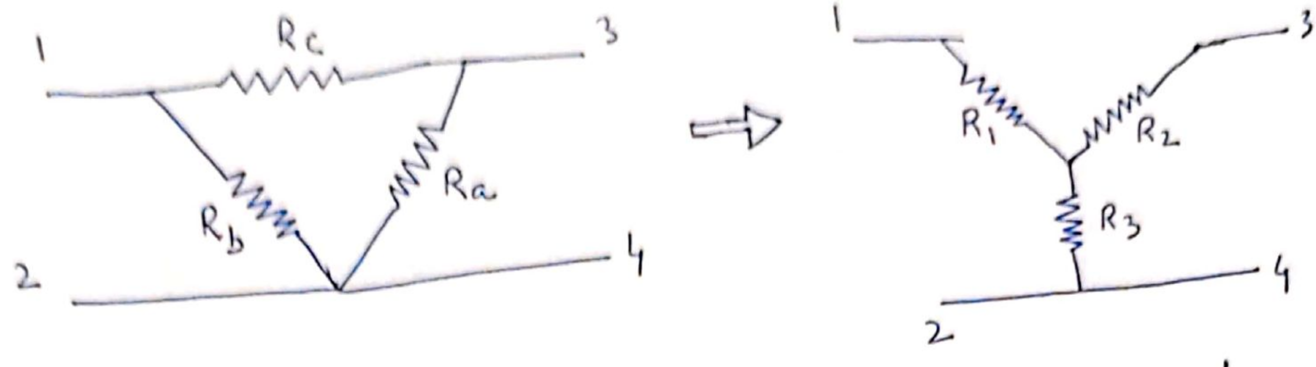


$(Z+R_5) \parallel (Y+R_6)$



$$\boxed{R_{eq} = R_1 + X + \frac{(Z+R_5)(Y+R_6)}{Z+R_5+Y+R_6}}$$

Delta to Wye Conversion



for the two networks to be equivalent of each other, the resistance between ~~any~~ each pair of nodes in Δ & Y are equal.

$$R_{12}(Y) = R_{12}(\Delta)$$

$$R_1 \overset{\text{in series}}{} R_3 = R_b \parallel (R_c + R_a)$$

$$R_1 + R_3 = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c} \quad \text{———} \quad (1).$$

Again,
$$(R_{13})(Y) = R_{13}(\Delta)$$

$$R_1 + R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+R_c} \quad \text{———} \quad (2).$$

$$R_{34}(Y) = R_{34}(\Delta)$$

$$R_2 + R_3 = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} \quad \text{———} \quad (3).$$

Add ①, ②, & ③, we get,

$$2(R_1+R_2+R_3) = \frac{2(R_aR_b+R_bR_c+R_cR_a)}{R_a+R_b+R_c}$$

or $$R_1+R_2+R_3 = \frac{R_aR_b+R_bR_c+R_cR_a}{R_a+R_b+R_c} \quad — ④.$$

Substract ① from ④, we get,

$$\cancel{R_1}+R_2+\cancel{R_3}-\cancel{R_1}-\cancel{R_3} = \frac{R_aR_b+R_bR_c+R_cR_a}{R_a+R_b+R_c} - \frac{R_aR_b-R_bR_c}{R_a+R_b+R_c}$$

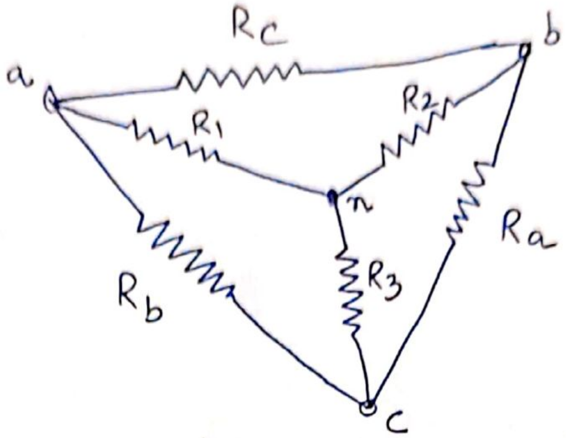
$$\boxed{R_2 = \frac{R_cR_a}{R_a+R_b+R_c}}$$

Similarly substract ② & ③ from ④, we get,

$$\boxed{R_3 = \frac{R_aR_b}{R_a+R_b+R_c}} \quad \& \quad \boxed{R_1 = \frac{R_bR_c}{R_a+R_b+R_c}}$$

Thus Δ- resistances have been represented in terms of equivalent Y- resistances which ensures Delta into Y- Conversion.

How to Remember this transformation?



We Create an extra node n for converting Δ into Y.

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}; \quad R_2 = \frac{R_a R_c}{R_a + R_b + R_c}; \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c};$$

Each resistor in the Y- network is the product of the resistors in the two adjacent Δ branches, divided by the sum of three Δ- resistors.

Assignment :-

Derive the set of equations for converting a Y into equivalent Δ network.